1.REAL NUMBERS

1. **Ramu says, "If $\log_{10} x = 0$, the value of x=0". Do you agree with him? Give reason. (June 2019)**

Sol: $\log_{10}^{x} = 0 \Rightarrow x = 10^{o} \Rightarrow x = 1$ **=>** $\boldsymbol{\log_{10}^{x}} \neq \mathbf{0}$**,** I cannot agree with Ramu

2. **Find the value of $\log_{\sqrt{2}} 128$ (JUNE 2018)**

Sol: Log $\log_{\sqrt{2}} 128 = x$

$\left(\sqrt{2}\right)^{x} = 128 = 2^{7} = \left(\sqrt{2}\right)^{14}$

$\therefore$ bases are equal so power should be equal

$\therefore x = 14$

$\therefore \log_{\sqrt{2}}^{128} = 14$

3. **Expand $\log_{10} 385$ (JUNE 2018)**

Sol: $\log_{10} 385 = \log_{10} 5 \times 7 \times 11$

$= \log_{10}^{5} + \log_{10}^{7} + \log_{10}^{11}$

$(\because \log xyz = \log x + \log y + \log z)$

4. **Lalitha says that HCF and LCM of the numbers 80 and 60 are 20 and 120 respectively. Do you agree with her? Justify.**

Sol: Two number are 80 and 60

H.C.F = 20

$L.C.M = 120$ $H.C.F \times L.C.M =$ product of two number

$H.C.F \times L.C.M = 20 \times 120 = 2400$

Product of two number = 80 x 60 =4800

These two are not equal so I didn't agree with her

5. **Find the value of $\log_{\sqrt{2}} 256$ (JUNE 2018)**

Sol: $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{8}$

$\sqrt{2} = 2^{\frac{1}{2}}$

$\log_{\sqrt{2}} 256 = \log_{2^{\frac{1}{2}}} 2^{8}$

$\left(\because \log_{b^{n}}^{a^{m}} = \frac{m}{n} \log_{b} a\right)$

$= \frac{8}{\frac{1}{2}} \log_{2} 2 = 8 \times \frac{2}{1} \times 1$

$\therefore \log_{\sqrt{2}} 256 = 16$

6. Expand $\log x^3y^2z^4$ **(MAY-2022)**

Sol: $\log x^3y^2z^4$

$\Rightarrow \log x^3 + \log y^2 + \log z^4$

$\Rightarrow 3\log x + 2\log y + 4\log z$

7. Find the HCF and LCM of 90, 144 by prime factorization method (MARCH-2019)

Sol:

2	90
3	45
3	15
	5

2	144
2	72
2	36
2	18
3	9
	3

$\therefore 90 = 2\times3\times3\times5 \; \therefore 144 = 2\times2\times2\times2\times3\times3$

$HCF = 2\times3\times3 = 18$

$LCM = 2\times3\times3\times5\times2\times2\times2 = 720$

8. Is $\log_3 81$ **rational or irrational? Justify your answer. (JUNE 2017)**

Sol: $\log_3 81 = \log_3 3^4$

$= 4\log_3 3 = 4\times1 = 4$

$\therefore \log_3 81 = 4$

$\therefore \log_3 81$ is a rational number

9. Write any two three-digit numbers. Find their L.C.M and G.C.D by prime factorization method.(MARCH 2017)

Sol: Two three-digit numbers are 120 and 150.

2	120
2	60
2	30
3	15
	5

2	150
3	75
5	25
	5

$120 = 2\times2\times2\times3\times5$
$150 = 2\times3\times5\times5$
$G.C.D = 2\times3\times5 = 30$
$L.C.M = 2\times3\times5\times2\times2\times5 = 600$

10. Prove that $2+\sqrt{3}$ is irrational (JUNE 2017)

Sol: let us suppose that $2+\sqrt{3}$ is rational let $2+\sqrt{3}=\frac{a}{b}$

Where a, b are integers $b\neq0$

$\sqrt{3}=\frac{a}{b}-2$

$\sqrt{3}=\frac{a-2b}{b}$

$\frac{a-2b}{b}$ is rational but $\sqrt{3}$ is irrational

$\therefore$ Our assumption is wrong

Hence $2+\sqrt{3}$ is irrational

11. Show that

$\log\frac{162}{343}+2\log\frac{7}{9}-\log\frac{1}{7}=\log2$ **(MARCH 2018)**

Sol: $L.H.S=\log\frac{162}{343}+2\log\frac{7}{9}-\log\frac{1}{7}$

$=\log\left(\frac{3^4\times2}{7^3}\right)+\log\frac{7^2}{9^2}-\log\frac{1}{7}$

$=\log(3^4\times2)-\log7^3+\log7^2-\log(3^2)^2$

$-(\log1-\log7)$

$=\log3^4+\log2-3\log7+2\log7-\log3^4$

$-\log1+\log7$

$=4\log3+\log2-3\log7+2\log7-4\log3+0$

$+\log(7)(\because\log1=0)$

$\Rightarrow\log2=RHS$

$\therefore L.H.S=R.H.S$

12. If $x^2+y^2=10xy$, prove that $2\log(x+y)=\log x+\log y+2\log2+\log3$.

Sol: $x^2+y^2=10xy$ **(MARCH 2016)**

Adding 2xy on both sides

$$x^2+y^2+2xy=10xy+2xy$$

$$(x+y)^2=12xy$$

Taking logarithms on both sides

$$\log(x+y)^2=\log 12xy$$

$$2\log(x+y)=\log 12+\log x+\log y$$

$$=\log 2^2\times 3+\log x+\log y$$

$$\log 2^2+\log 3+\log x+\log y$$

$$\therefore 2\log(x+y)=2\log 2+\log 3+\log x+\log y$$

13. If $x^2+y^2=27xy,$ **then show that** $\log\left(\frac{x-y}{5}\right)=\frac{1}{2}[\log x+\log y]$ **(JUNE 2017)**

Sol: $x^2+y^2=27xy$

Subtract 2xy on both sides

$$x^2+y^2-2xy=27xy-2xy$$

$$(x-y)^2=25xy$$

$$\Rightarrow \frac{(x-y)^2}{25}=xy$$

$$\Rightarrow \left(\frac{x-y}{5}\right)^2=xy$$

Square roots on both sides

$$\Rightarrow \sqrt{\left(\frac{x-y}{5}\right)^2}=\sqrt{xy}$$

$$\Rightarrow \frac{(x-y)}{5}=(xy)^{\frac{1}{2}}$$

Apply logarithm on both sides

$$\Rightarrow \log\frac{x-y}{5}=\log(xy)^{\frac{1}{2}}$$

$$\Rightarrow \log\left(\frac{x-y}{5}\right)=\frac{1}{2}(\log xy)$$

$$\Rightarrow \log\left(\frac{x-y}{5}\right)=\frac{1}{2}(\log x+\log y)$$

14.Show that $\sqrt{5}-\sqrt{3}$ is an irrational number. (JUNE 2019)

Sol: Let $\sqrt{5}-\sqrt{3}$ is an irrational number

$$\therefore \sqrt{5}-\sqrt{3}=\frac{p}{q}$$

Squaring on both sides

$$\left(\sqrt{5}-\sqrt{3}\right)^2=\left(\frac{p}{2}\right)^2$$

$$\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2-2\sqrt{5}.\sqrt{3}=\frac{p^2}{q^2}$$

$$\left(\therefore (a-b)^2=a^2-2ab+b^2\right)$$

$$5+3-2\sqrt{15}=\frac{p^2}{q^2}$$

$$8-\frac{p^2}{q^2}=2\sqrt{15}$$

$$\sqrt{15}=\frac{8q^2-p^2}{2q^2}$$

It is contradiction our assumption is false Hence $\sqrt{5}-\sqrt{3}$ is an irrational number

15.Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number. (MARCH 2018)

Sol: Suppose $\sqrt{3}+\sqrt{5}$ is not an irrational number

Then $\sqrt{3}+\sqrt{5}$ must be a rational number

$\sqrt{3}+\sqrt{5}=\frac{p}{q}, q\neq 0$ and $p,q\in Z$

Squaring on both side

$$\left(\sqrt{3}+\sqrt{5}\right)^2=\left(\frac{p}{q}\right)^2$$

$$3+5+2\sqrt{15}=\frac{p^2}{q^2}$$

$$8+2\sqrt{15}=\frac{p^2}{q^2}$$

$$2\sqrt{15}=\frac{p^2}{q^2}-8=\frac{p^2-8q^2}{q^2}$$

$\sqrt{15}=\frac{p^2-8q^2}{2q^2}$

But $\sqrt{15}$ is an irrational number $\frac{p^2-8q^2}{2q^2}$ is a rational number

But an irrational number cannot be equal to a rational number

16. Prove that $\sqrt{2}+\sqrt{11}$ is an irrational number. (MARCH 2019)

Sol: let us suppose that $\sqrt{2}+\sqrt{11}$ is rational $\therefore \sqrt{2}+\sqrt{11}=\frac{a}{b}$ (where a, b are integers and $b\neq 0$)

Squaring on both sides, we get

$\left(\sqrt{2}+\sqrt{11}\right)^2=\left(\frac{a}{b}\right)^2$

$2+11+2\sqrt{22}=\frac{a^2}{b^2}$

$2\sqrt{22}=\frac{a^2}{b^2}-13=\frac{a^2-13b^2}{b^2}$

$\sqrt{22}=\frac{a^2-13b^2}{2b^2}$

Since a, b are integers $\frac{a^2-13b^2}{2b^2}$ is rational and so $\sqrt{22}$ is rational

This contradicts the fact that $\sqrt{22}$ is irrational Hence $\sqrt{2}+\sqrt{11}$ is irrational

17. Prove that $\sqrt{2}+\sqrt{7}$ is an irrational number. (MAY 2022)

$\sqrt{2}+\sqrt{7}=\frac{p}{q}$ (where p, q are integers)

Squaring on both sides

$\left(\sqrt{2}+\sqrt{7}\right)^2=\left(\frac{p}{q}\right)^2$

$\left(\sqrt{2}\right)^2+\left(\sqrt{7}\right)^2+2\times\sqrt{2}\times\sqrt{7}=\left(\frac{p}{q}\right)^2$

$\left[\because (a+b)^2=a^2+b^2+2ab\right]$

$2+7+2\sqrt{14}=\frac{p^2}{q^2}$

$9+2\sqrt{14}=\frac{p^2}{q^2}$

$$2\sqrt{14} = \frac{p^2 - 9q^2}{q^2}$$

$$\sqrt{14} = \frac{p^2 - 9q^2}{2q^2}$$

It contradicts that $\sqrt{14}$ is an irrational number

Out assumption is false

Hence $\sqrt{2} + \sqrt{7}$ is an irrational number

18. Use division algorithm to show that the square of any positive integer is of the form 5m or 5m+1 or 5m+4, where 'm' is a whole number. (JUNE 2018)

Sol: Euclid's division lemma:

$a = bq + r \text{ and } 0 \le r < b$

$\therefore$ a values are

$5q + 0, 5q + 1, 5a + 2, 5q + 3, 5q + 4$

Step-I: a=5q

Squaring on both sides

$a^2 = (5q)^2 = 25q^2 = 5(5q^2) = 5m$

$(\because 5q^2 \in w)$

Step -2: $a = 5q + 1$

Squaring on both sides

$a^2 = (5q+1)^2 = 25q^2 + 10q + 1$

$= 5(5q^2 + 2q) + 1 = 5m + 1$

$(\because 5q^2 + 2q \in W)$

Step -3: $a = 5q + 2$

Squaring on both sides

$a^2 = (5q+2)^2 = 25q^2 + 20q + 4$

$= 5(5q^2 + 4q) + 4 = 5m + 4$

$(\because 5q^2 + 4q \in W)$

Step -4: $a = 5q + 3$

Squaring on both sides

$a^2 = (5q+3)^2 = 25q^2 + 30q + 9$

$= 25q^2 + 30q + 5 + 4$

$= 5(5q^2 + 6q + 1) + 4$

$= 5m + 4 (\because 5q^2 + 6q + 1 = m \in W)$

Step -5: $a = 5q + 4$

Squaring on both sides

$a^2 = (5q+4)^2 = 25q^2 + 40q + 16$

$= 25q^2 + 40q + 15 + 1$

$= 5(5q^2 + 8q + 3) + 1$

$= 5m + 1 (\because 5q^2 + 8q + 3 = m \in W)$

$\therefore$ The square of any positive integer is of the form 5m or 5m+1 or 5m+4

19. Show that cube of any positive integer will be in the form of 8m or 8m+1 or 8m+3 or 8m+5 or 8m+7, where m is a whole number. (MARCH 2018)

Sol: $a = bq + r, 0 \le r \le b$

$a = 8k + t \text{ for } t = 0,1,2,3,4,5,6,7$

$a^3 = (8k+t)^3$

$= (8k)^3 + 3(8k)^2(t) + 3(8k)t^2 + t^3$

$= 8(64k^3 + 24k^2t + 3t^2k) + t^3$

$a^3 = 8n + t^3$

If $t = 0 \Rightarrow 8n + 0^3 = 8n = 8m$

$t = 1 \Rightarrow 8n + 1^3 = 8n + 1 = 8m + 1$

$t = 2 \Rightarrow 8n + 2^3 = 8n + 8 = 8(n+1) = 8m$

$t = 3 \Rightarrow 8n + 3^3 = 8n + 27 =$
$8(n+3) + 3 = 8m + 3$

$t = 4 \Rightarrow 8n + 4^3 = 8n + 64$
$= 8(n+8) = 8m$

$t = 5 \Rightarrow 8n + 5^3 = 8n + 125$
$= 8(n+15) + 5 = 8m + 5$

$t = 6 \Rightarrow 8n + 6^3 = 8n + 216$
$= 8(n+27) = 8m$

$t = 7 \Rightarrow 8n + 7^3 = 8n + 343$

$= 8(n + 42) + 7 = 8m + 7$

$\therefore$ The cube of any positive integer will be of the form 8m (or) 8m+1 (or) 8m+3 (or) 8m+5 (or) 8m+7

2.SETS

1.List all its subsets of the set $A = \{x, y, z\}$

Sol: Given that $A = \{x, y, z\}$ then n(A) = 3 then number of subsets are $2^3 = 8$

$\therefore$Subsets of Set A are $\phi, \{x\}, \{y\}, \{z\}, \{x, y\}$, $\{x, y\}, \{y, z\}, \{z, x\}, \{x, yz\}$

2. Give one example each for a finite set and an infinite set. (MARCH 2018)

Sol: finite set : $A = \{2, 3, 5, 7\}$

Infinite set : $A = \{1, 3, 5, 7, ...\}$

3. If $A=\{1,2,3,5\}, B=\{3,4,5,6\}$ **find** $A \cap B$**. (JUNE 2017)**

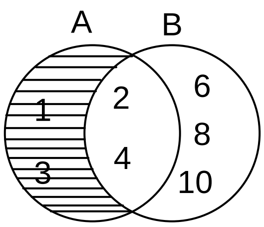
Sol: Given $A=\{1,2,3,5\}, B=\{3,4,5,6\}$

$A \cap B=\{1,2,3,5\} \cap \{3,4,5,6\}$

$\therefore A \cap B=\{3,5\}$

4.If $A=\{1,2,3,4\}, B=\{2,4,6,8,10\}$**, then represent the Vennu diagram of A-B.**

Sol: Given $A=\{1,2,3,4\}, B=\{2,4,6,8,10\}$



A-B = ▤

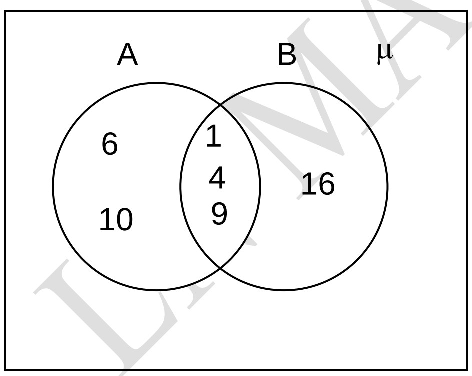
$\therefore A-B=\{1,3\}$

5. Represent $A \cap B$ **through Venn diagram, where** $A=\{1,4,6,9,10\}$ **and B = {perfect squares less than 25}**

Sol: Given $A=\{1,4,6,9,10\}$

B = {perfect squares less than 25}

$\therefore B=\{1,4,9,16\}$



$\therefore A \cap B=\{1,4,9\}$

6.If $A=\{x: x \in N, x<10\}, B=\{x: x \text{ is a prime number and } x<10\}$

then show that $A-B \neq B-A$ **with the help of Venn diagram. (JUNE 2017)**

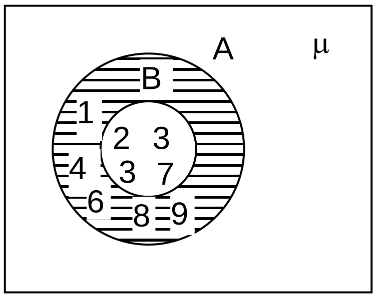
Sol: $A=\{x:x\in N, x<10\} \Rightarrow A=\{1,2,3,4,5,6,7,8,9\}$

$B=\{x: x \text{ is a prime number and } x<10\} \Rightarrow B=\{2,3,5,7\}$

Then $A=\{1,2,3,4,5,6,7,8,9\}$ - $B=\{2,3,5,7\}$

$A-B=\{1,4,6,8,9\} \Rightarrow B-A=\phi\,(\because B\subset A)$

$\therefore A-B\neq B-A$



7. If $A=\{1,2,3\}, B=\{3,4,5\}$ **, then find A-B and B-A**

Sol: Given $A=\{1,2,3\}, B=\{3,4,5\}$

$A-B=\{1,2,3\}-\{3,4,5\}=\{1,2\}$

$B-A=\{3,4,5\}-\{1,2,3\}=\{4,5\}$

$\therefore A-B=\{1,2\}, B-A=\{4,5\}$

8. If $A=\{x: x \text{ is a factor of } 12\}$ **and** $B=\{x: x \text{ is a factor of } 6\}$ **, then find** $A\cup B$ and $A\cap B$ **(Marh2016)**

Sol:

$A=\{x: x \text{ is factor of } 12\} \Rightarrow A=\{1,2,3,4,6,12\}$

$B=\{x: x \text{ is factor of } 6\} \Rightarrow B=\{1,2,3,6\}$

$A\cup B=\{1,2,3,4,6,12\}\cup\{1,2,3,6\}$

$\therefore A\cup B=\{1,2,3,4,6,12\}$

$A\cap B=\{1,2,3,4,6,12\}\cap\{1,2,3,6\}$

$\therefore A\cap B=\{1,2,3,6\}$

9. If $A=\{x: x \text{ is a prime less than } 20\}$ **and** $B=\{x: x \text{ is a whole number less than } 10\}$ **, then verify** $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

Sol: $A=\{x: x \text{ is a prime number } x<20\} \Rightarrow A=\{2,3,5,7,11,13,17,19\}$

$B=\{x: x \text{ is an integer } x<10\} \Rightarrow B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cup B = \{2,3,5,7,11,13,17,19\} \cup$
$\{0,1,2,34,5,6,7,8,9\}$
$= \{0,1,2,3,4,5,6,8,9,11,13,17,19\}$
$A \cap B = \{2,3,5,7,11,13,17,19\} \cap$
$\{0,1,2,3,4,5,6,7,8,9\}$
$= \{2,3,5,7\}$

Then n(A) = 8, n(B) = 10, then n(AUB) = 14, $n(A \cap B) = 4$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

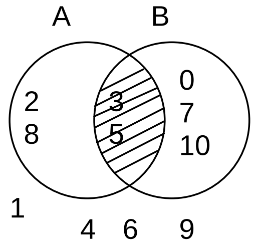
=> $14 = 8 + 10 - 4$

$14 = 14$

10.If $\mu = \{0,1,2,3,4,5,6,7,8,9,10\}$ **,** $A = \{2,3,5,8\}$ **and** $B = \{0,3,5,7,10\}$**, then represent** $A \cap B$ **in the Venn diagram.**

Sol: Given $\mu = \{0,1,2,3,4,5,6,7,8,9,10\}$

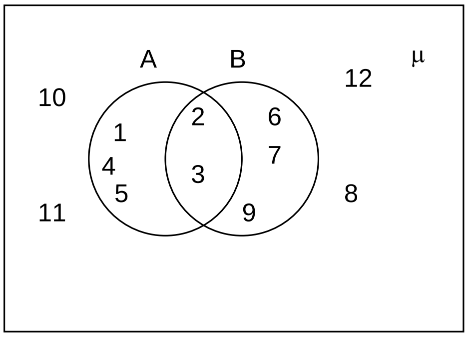
$A = \{2,3,5,8\}$, $B = \{0,3,5,7,10\}$



$\therefore A \cap B = \{3,5\} =$ ▨

11. Using the Venn diagram, verify $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
(JUNE 2017)

Sol:



$n(A\cup B)=8$

$(\because A\cup B)$

$=\{1,2,3,4,5,6,7,9\}$

$n(A)=5(\because A=\{1,2,3,4,5\})$

$n(B)=5(\because B=\{2,3,6,7,9\})$

$n(A\cap B)=2(\because A\cap B=\{2,3\})$

$n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$8=5+5-2$

$8=8$

12. $A=\{x,2x+1,x\in N,x\le 5\}$ $B=\{x/x \text{ is compositie number}, x\le 12\}$ **,Then show that** $(A\cup B)=(A\cap B)=(A-B)\cup(B-A)$

Sol: $A=\{2x+1;x\in N,x\le 5\}$

$A=\{3,5,7,9,11\}$

$B=\{x: x \text{ is a composite number } x\le 12\}$

$A\cup B=\{3,5,7,8\}\cup\{4,6,8,9,10,12\}=\{3,4,5,6,7,8,9,10,11,12\}$

$A\cap B=\{3,5,7,9,11\}\cap\{4,6,8,9,10,12\}=\{9\}$

$(A\cup B)-(A\cap B)=\{3,4,5,6,7,8,9,10,11,12\}-\{9\}$

$=\{3,4,5,6,7,8,10,11,12\}$

$A-B=\{3,5,7,9,11\}-\{4,6,8,9,10,12\}=\{3,5,7,11\}$

$B-A=\{4,6,8,9,10,12\}-\{3,5,7,9,11\}=\{4,6,8,10,12\}$

$(A-B)\cup(B-A)$

$=\{3,5,7,11\}\cup\{4,6,8,10,12\}$

$=\{3,4,5,6,7,8,10,11,12\}$

$\therefore(A\cup B)-(A\cap B)=(A-B)\cup(B-A)$

13. **If** $A = \{x : x \text{ is a prime and } x < 10\}$

$B = \{x : x \text{ is a factor of } 6\}$ **then find** $A \cap B, A \cup B$ **and** $A - B$

Sol: $A = \{x : x \text{ is a prime and } x < 10\}$ **=>** $A = \{2,3,5,7\}$

$B = \{x : x \text{ is a factor of } 6\}$ **=>** $B = \{1,2,3,6\}$

$A \cap B = \{2,3,5,7\} \cap \{1,2,3,6\} = \{2,3\}$

$A \cup B = \{2,3,5,7\} \cup \{1,2,3,6\}$
$= \{1,2,3,5,6,7\}$

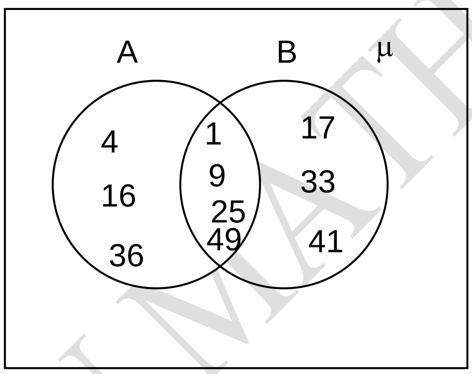
$A - B = \{2,3,5,7\} - \{1,2,3,6\} = \{5,7\}$

14. A = {x : x is a perfect square, x<50, $x \in N$**}**
B= {x : x = 8m+1, where $m \in W, x < 50, x \in N$**}**
Find the A<B and display it with Venn diagram (MARCH 2018)

Sol: A = {x:x is a perfect square, x<50, $x \in N$} **=>** $A = \{1,4,9,16,25,36,49\}$

B = {x:x=8m+1, where $m \in W, x < 50, x \in N$} **=>** $B = \{1,9,17,25,33,41,49\}$

$\therefore A \cap B = \{1,9,25,49\}$



$\therefore A \cap B = \{1,9,25,49\}$

FOR MORE CHAPTERS ..

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